

# **CS09 304 Discrete Computational Structures**

**Question Bank – Module III- Group Theory**

Topic	Question	Mark	Month & Year	Regulation
Sub groups, Abelian Groups, Cosets	Show that $x*y = x - y$ is not a binary operation over the set of natural numbers but it is a binary operation on the set of integers.	7	June 2009	04
	Prove that G is an abelian group if and only if $(a.b)^2 = a^2.b^2$ for all $a,b \in G$	5	Oct 2011	09
	Prove that G is an abelian group if and only if $(a.b)^{-1} = a^{-1}.b^{-1}$ for all $a,b \in G$	5	Dec 2010	09
	If $(G,*)$ is a group of even order prove that it has an element $a \neq e$ satisfying $a^2=e$	8	Dec 2007	04
	Show that $Z_7=(\{1,2,3,4,5,6\},*\text{mod}7)$ is an abelian group	5	Oct 2011 Nov 2012	09
	Show that $Z_4$ forms a group with respect to the operation, addition modulo 4	7	Dec 2009	04
	Let S be the set of real numbers except -1. Define * on S by $a*b = a+b+ab$ . Show that $(S,*)$ is an abelian group.	10	Oct 2011	09
	Show that the set $Q^+$ of all positive rational numbers forms an Abelian group under the operation defined by $a * b = \frac{ab}{2}$ where $a,b \in Q^+$	8	June 2009	04
	Define coset of a subgroup and show that for any subgroup H of a group G both right and left cosets of H have same number of elements.	5	Dec 2005	04
	Show that the set $G = \{-1,1\}$ is a finite abelian group of order 2 under multiplication.	7	Dec 2007	04
	Let G be a group of $2 \times 2$ matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where $a,b,c,d$ are real numbers such that $ad-bc \neq 0$ . Show that G is a group under matrix multiplication. Let $H = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$ where $a,b,d$ are real numbers such that $ad \neq 0$ . Show that H is a subgroup.	10	Oct 2011 Nov 2012	09
Let $G = \{(a,b): a \neq 0, b \in \mathbb{R}\}$ and * be the binary composition defined by $(a,b) * (c,d) = (ac, bc+d)$ Show that $(G,*)$ is a group. Also show that it is non abelian	10	Dec 2010	09	
Cyclic Groups	Every subgroup of a cyclic group is cyclic	5	Dec 2001	2k
	Show that every cyclic group is abelian	3	Dec 2006 Dec 2007	04

## CS09 304 Question Bank for Module III(Group Theory)

	Let $G$ be a finite cyclic group of order 60 with generator $g$ . Find the order of the subgroup generated by $g^{25}$	5	Dec 2010	09
	Prove that if $G$ is a finite group whose order is a prime number $p$ then $G$ is a cyclic group	5	Dce 2010	09
<b>Lagrange's Theorem</b>	State and prove Lagrange's theorem	5	Dec 2008 Dec 2009 Dec 2010 Nov 2012	09
	State and prove Lagrange's theorem with necessary results	15	Dec 2005	04
<b>Homomorphism</b>	Explain homomorphism function	2	Dec 2010	09
	Show that group homomorphism preserves identity, inverse and subgroup.	5	Nov 2012	09
	Let $(G,*)$ be a group and $a \in G$ . Let $f:G \rightarrow G$ be given by $f(x) = a*xa^{-1}$ for every $x \in G$ . Prove that $f$ is an isomorphism of $G$ onto $G$	10	Nov 2012	09
	Let $R$ be the group of real numbers under addition and $R^+$ be the group of positive real numbers under multiplication. Let $f: R \rightarrow R^+$ be defined by $f(x) = e^x$ . Then show that $f$ is an isomorphism.	7	Dec 2008	04
<b>Coding Theory</b>	Prove that the minimum weight of the non zero code words in a group code is equal to its minimum distance.	8	Dec 2009	04
	Define generator matrices	2	Nov 2012	09
	Define Hamming Code	2	Dec 2009 Oct 2011	04,09
	Define group code	5	Dec 2008	04
	Show that $(2,5)$ encoding function defined by $e(00) = 00000$ , $e(01) = 01110$ , $e(10) = 10101$ , $e(11) = 11011$ is a group code	8	Dec 2008	04