C 40919 (Pages : 2) Name.....................................

Reg. No.................................

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, APRIL 2013

EN/PTEN 09 101—ENGINEERING MATHS–I

(2009 Scheme)

[Regular/Supplementary/Improvement]

Time : Three Hours Maximum : 70 Marks

Part A

Answer **all** *questions.*

1. Give the centre of curvature formula in cartesian form.
2. What is meant by Absolute convergence ? Define.
3. Test for convergence the series $\Sigma\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$.
4. Prove that the eigen values of real symmetric matrix are real.
5. Express $f(x) = x$ as a half-range cosine series in $0 < x < 2$.

(5 × 2 = 10 marks)

Part B

Answer any **four** *questions.*

6. Test for convergence the series $\frac{1}{1\cdot2\cdot3}+\frac{3}{2\cdot3\cdot4}+\frac{5}{3\cdot4\cdot5}+\ldots\ldots\infty$.
7. Discuss the convergence of $\sum_{n=1}^{\infty}\frac{n^2}{3^n}$.
8. Find the radius of curvature at the point $(a\cos^3\theta, a\sin^3\theta)$ on the curve $x^{2/3}+y^{2/3}=a^{2/3}$.
9. Find the minimum value of $x^2+y^2+z^2$, when $x+y+z=3a$.
10. Find the eigen values of $A=\begin{bmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{bmatrix}$.
11. Find the Fourier series expansion for $f(x)$, if $f(x)=e^{-x}$ in $0<x<2\pi$.

(4 × 5 = 20 marks)

Turn over

Part C

Answer Section (a) **or** *Section (b) of each question.*

12. (a) Find the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 if $y_1 = \frac{x_2 x_3}{x_1}$, $y_2 = \frac{x_3 x_1}{x_2}$, $y_3 = \frac{x_1 x_2}{x_3}$.

Or

(b) Given the transformation $u = e^x \cos y$ and $v = e^x \sin y$ and that f is a function of u and v and also of x and y, prove that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = (u^2 + v^2)\left(\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2}\right)$.

13. (a) State the value of x for which the following series converges $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \ldots\ldots\ldots\infty$ converges.

Or

(b) Test the series for convergence

$$1 + \frac{a}{b}x + \frac{a(a+1)}{b(b+1)}x^2 + \frac{a(a+1)(a+2)}{b(b+1)(b+2)}x^3 + \ldots\ldots + \infty \; (a > 0, b > 0, x > 0).$$

14. (a) Verify Cayley–Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and also use it to find A^{-1}.

Or

(b) Determine the nature of the following quadratic forms without reducing them to canonical forms $x_1^2 + 3x_2^2 + 6x_3^2 + 2x_1 x_2 + 2x_2 x_3 + 4x_3 x_1$.

15. (a) Expand $f(x) = x \sin x, 0 < x < 2\pi$, in a Fourier series.

Or

(b) Find the Fourier series expansion for $f(x)$, if

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots.. = \frac{\pi^2}{8}$.

(4 × 10 = 40 marks)