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Reg. No.........

THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2009

CS/IT 04 303—DISCRETE COMPUTATIONAL STRUCTURES

(2004 admissions)

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

1. (a) Show that the statement $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a tautology.

   (b) Show that $P(x) \wedge (x)\,Q(x) \Rightarrow (\exists x)\left(P(x) \wedge Q(x)\right)$.

   (c) Define a relation on a set and give an example.

   (d) If $f : R \to R$ is given by $f(x) = 3x - 7$, find f^{-1}.

   (e) Define Hamming code.

   (f) Give an example of subring.

   (g) Find all the subgroups of a group G of prime order.

   (h) Define a field.

   (8 × 5 = 40 marks)

2. (a) (i) Show that $\left(\neg P \wedge (\neg Q \wedge R)\right) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$. (7 marks)

   (ii) Obtain the principal conjunctive normal form of the formula :

   $$(\neg P \to R) \wedge (Q \leftrightarrow R)$$

   (8 marks)

   Or

   (b) (i) Without using truth table, find principal disjunctive normal form of

   $$(P \wedge Q) \vee (\neg P \wedge Q) \vee (Q \wedge R)$$

   (7 marks)

   (ii) Show that if $p \to q$, $q \to r$, $\neg(p \wedge r)$ and $p \vee r$ then r. (8 marks)

3. (a) (i) If R is the relation on the set of ordered pairs of positive integers such that $(a, b), (c, d) \in R$ whenever $ad = bc$, show that R is an equivalence relation.

   (7 marks)

   (ii) If $A = \left\{x \in R / x \neq \frac{1}{2}\right\}$ and $f : A \to R$ is defined by $f(x) = \dfrac{4x}{2x - 1}$, find the range of f.

   (8 marks)

   Or

Turn over

(b) (i) If R is the relation on the set of integers such that $(a, b) \in R$ iff $b = a^m$ for some positive integer m, show that R is a partial ordering.

(7 marks)

(ii) If R and S be relations on a set A represented by the matrices

$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \quad M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

find the matrices that represent SOR and ROS.

(8 marks)

4. (a) (i) Show that the set $Z_4 = \{[0], [1], [2], [3]\}$ forms a group with respect to $+_4$. (7 marks)

(ii) Show that the algebraic system $(N, +)$ and $(Z_4, +_4)$ are homomorphic. (8 marks)

Or

(b) (i) State and prove Lagrange's Theorem. (7 marks)

(ii) Prove that the minimum weight of the non-zero code words in a group code is equal to its minimum distance.

(8 marks)

5. (a) (i) Show that every homomorphic image of a commutative ring is commutative.

(7 marks)

(ii) Prove that an arbitrary intersection of subrings is a subring. (8 marks)

Or

(b) (i) Show that the set of numbers of the form $a + b\sqrt{2}$ with a and b as rational numbers is a field.

(7 marks)

(ii) If R is an arbitrary ring and R′ is the set of constant polynomials in R[x], then R′ is isomorphic to R.

(8 marks)

[4 × 15 = 60 marks]