

D 2312

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Name.....

Reg. No.....

**THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE
 EXAMINATION, DECEMBER 2009**

EN 04-301-A—ENGINEERING MATHEMATICS

(Common to all except CS and IT)

(2004 admissions)

Time : Three Hours.

Maximum : 100 Marks

Answer all questions.

Part A

1. (a) Show that if two of the vectors V_1, \dots, V_m are equal, say $V_1 = V_2$, then vectors are linearly dependent.
- (b) Show that two vectors V_1 and V_2 are dependent if and only if one of them is a multiple of the other.
- (c) Expand the (f_n) function $f(x) = \sin x, 0 \leq x \leq \pi$ in Fourier cosine series.
- (d) Prove that if $F'(x) = F(x)$, then $F(x) = e^x + C$, where C is a constant.
- (e) If a random variable has a Poisson distribution such that $P(1) = P(2)$, find the mean of the distribution.
- (f) A variate X has the probability distribution

x	1	4	6	9
$P(X = x)$	$1/6$	$1/2$	$1/3$	
- (g) What is the use of goodness of fit test? (X²-test of goodness of fit).
- (h) A die is tossed 960 times and it falls with 5 upwards 184 times. Is the die biased?

(8 + 5 = 10 marks)

Part B

2. (a) (i) Consider the following basis of Euclidean space \mathbb{R}^3 : $\{V_1 = (1, 1, 1), V_2 = (0, 1, 1), V_3 = (0, 0, 1)\}$. Use the Gram Schmidt algorithm to transform $\{V_i\}$ into an orthonormal basis $\{U_i\}$ of \mathbb{R}^3 . (8 marks)
 - (ii) Find the matrix A of the inner product with respect to the basis $\{1, t, t^2\}$ of V . (7 marks)
- Or
- (b) (i) Let $F : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the linear mapping defined by

$$F(x, y, z) = (x + y + z, x + 2z, x + y + 3z)$$
 Find a basis and the dimension of the image U of F . (8 marks)

(8 marks)

Turn over

(ii) Find $\cos \theta$ for the angle θ between $f(t) = 2t^2 - 1$ and $g(t) = t^3$ in the inner space V of polynomials with inner product $\langle f, g \rangle = \int_0^1 f(t)g(t) dt$.

(7 marks)

10. (ii) Find the Fourier transform of $f(x) = \frac{1-x}{2} \frac{e^{-x/2}}{1+x^2}$. Hence evaluate $\int_0^{\infty} \cos \frac{x}{2} dy$.

(ii) (a) Expand $f(x) = x \sin x$ as a cosine series in $0 < x < \pi$. (8 marks)

(b) In $-\pi < x < \pi$, express $\sin x$ as a Fourier series of periodicity 2π . (7 marks)

11. (ii) (a) If X and Y are independent binomial random variables having respective parameters (n, p) and (m, p) . Prove that the conditional probability mass function of X , given that $X + Y = k$, is that of a hypergeometric random variable. (8 marks)

(a) In a referendum 60% of voters voted in favour. A random sample of 200 voters was selected. What is the probability that in the sample (a) more than 130 voted in favour? (b) between 105 and 130 inclusive voted in favour? (7 marks)

(ii) (a) In a normal distribution, 91% of the items are under 15 and 8% are over 64. Find the mean and S.D. of the distribution. (8 marks)

(a) The probability that a pen manufactured by a company will be defective is $\frac{1}{10}$. If 12 such pens are manufactured, find the probability that (a) exactly two will be defective; (b) at least two will be defective; (c) none will be defective. (7 marks)

12. (ii) (a) 15% of a random sample of 1800 undergraduates were smokers. Whereas 20% of a random sample of 2000 post-graduates were smokers in a state. Can we conclude that less number of undergraduates are smokers than the post-graduates? (8 marks)

(a) In two large populations there are 40% and 23% respectively of married people. Is this difference likely to be hidden in samples of 1200 and 900 respectively from the two populations? (7 marks)

(b) (i) 400 children are chosen in an industrial town and 150 are found to be under weight. Assuming the conditions of simple sampling, estimate the percentage of children who are under weight in the industrial town and assign limits within which the percentage probably lies? (8 marks)

(ii) A sample of 1000 days is taken from meteorological records of a certain district and 120 of them are found to be foggy. What are the probable limits to the percentage of foggy days in the district? (7 marks)

[1 + 15 = 160 marks]