Module V

Answer any three questions, each carries 5 marks.

19 Find the work done by $F(x,y) = (x^2 + y^2)i - xj$ along the curve $C: x^2 + y^2 = 1$ counter clockwise from $(1,0)$ to $(0,1)$ (5)

20 Determine whether $F(x,y) = 6y^2\, i + 12xy\, j$ is a conservative vector field. If so find the potential function for it. (5)

21 Find the divergence and curl of the vector field $F(x,y,z) = xyz^2 i + yzx^2\, j + zxy^2\, k$ (5)

22 Prove that $\int_C (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}\, .\, d\bar{r}$ is independent of the path and evaluate the integral along any curve from (0,0,0) to (1,2,3). (5)

23 If $\bar{r} = x\,\bar{i} + y\,\bar{j} + z\,\bar{k}$ and $r = \|\bar{r}\|$, prove that $\nabla^2 f(r) = \frac{2}{r} f'(r) + f''(r)$. (5)

Module VI

Answer any three questions, each carries 5 marks.

24 Using Green's theorem evaluate $\int_c (xy + y^2)dx + x^2\, dy$ where C is the boundary of the region bounded by $y = x^2$ and $x = y^2$ (5)

25 Evaluate the surface integral $\iint_\sigma z^2\ ds$, where σ is the portion of the curve $z = \sqrt{x^2 + y^2}$ between $z = 1$ and $z = 3$ (5)

26 Determine whether the vector field $F(x,y,z)$ is free of sources and sinks. If not, locate them. (5)

$i) F(x,y,z) = (y + z)\bar{i} - xz\bar{j} + x^2 \sin y\ \bar{k}$

$ii) F(x,y,z) = x^3\bar{i} + y^3\bar{j} + 2z^3\bar{k}$

27 Use divergence theorem to find the outward flux of the vector field $F(x,y,z) = (2x + y^2)i + xy\, j + (xy - 2z)k$ across the surface σ of the tetrahedron bounded by $x + y + z = 2$ and the coordinate planes. (5)

28 Using Stoke's theorem evaluate $\int_C \bar{F}\, .\, d\bar{r}$; where $\bar{F} = xy\,\bar{i} + yz\,\bar{j} + xz\,\bar{k}$; C triangular path in the plane $x + y + z = 1$ with vertices at $(1,0,0), (0,1,0)$ and (0,0,1) in the first octant (5)

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