

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100

Duration: 3 Hours

PART A

Answer all questions, each carries 5 marks.

- | | | Marks |
|---|--|-------|
| 1 | a) Determine whether the series $\sum_{k=0}^{\infty} \frac{5}{4^k}$ converges. If so, find the sum | (2) |
| | b) Examine the convergence of $\sum \left(\frac{k}{k+1}\right)^{k^2}$ | (3) |
| 2 | a) Find the slope of the surface $z = x e^{-y} + 5y$ in the y direction at the point (4, 0) | (2) |
| | b) Show the function $f(x, y) = e^x \sin y + e^y \cos x$ satisfies the Laplace's equation $f_{xx} + f_{yy} = 0$ | (3) |
| 3 | a) Find the directional derivative of $f(x, y, z) = x^3 z - yx^2 + z^2$ at P (2, -1, 1) in the direction of $3\vec{i} - \vec{j} + 2\vec{k}$ | (2) |
| | b) Find the unit tangent vector and unit normal vector to the curve $\mathbf{r}(t) = 4\cos t \mathbf{i} + 4 \sin t \mathbf{j} + t \mathbf{k}$ at $t = \frac{\pi}{2}$ | (3) |
| 4 | a) Using double integration, evaluate the area enclosed by the lines $x = 0, y = 0, \frac{x}{a} + \frac{y}{b} = 1$ | (2) |
| | b) Evaluate $\int_{-1}^2 \int_0^2 \int_0^1 (x^2 + y^2 + z^2) dx dy dz$ | (3) |
| 5 | a) If $\mathbf{F}(x, y, z) = x^2 \mathbf{i} - 3\mathbf{j} + yz^2 \mathbf{k}$ find $\text{div } \mathbf{F}$ | (2) |
| | b) Find the work done by the force field $\mathbf{F} = xy \mathbf{i} + yz \mathbf{j} + zx \mathbf{k}$ on a particle that moves along the curve C: $x = t, y = t^2, z = t^3, 0 \leq t \leq 1$ | (3) |
| 6 | a) Use Green's theorem to evaluate $\int_c (x dy - y dx)$, where c is the circle $x^2 + y^2 = a^2$ | (2) |
| | b) If S is any closed surface enclosing a volume V and $\mathbf{F} = x\mathbf{i} + 2y\mathbf{j} + 3z\mathbf{k}$ show that $\iint_S \mathbf{F} \cdot \mathbf{n} ds = 6V$ | (3) |

PART B

Module I

Answer any two questions, each carries 5 marks.

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|---|---|-----|
| 7 | Determine whether the alternating series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+7}{k(k+4)}$ is absolutely convergent | (5) |
| 8 | Find the Taylor series expansion of $f(x) = \frac{1}{x+2}$ about $x = 1$ | (5) |
| 9 | Find the interval of convergence and radius of convergence of $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{(x+1)^k}{k}$ | (5) |

Module II

Answer any two questions, each carries 5 marks.

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|----|--|-----|
| 10 | Find the local linear approximation L to the function $f(x, y, z) = xyz$ at the point P (1, 2, 3). Also compare the error in approximating f by L at the point Q (1.001, 2.002, 3.003) with the distance PQ. | (5) |
| 11 | Locate all relative extrema and saddle points of $f(x, y) = 2xy - x^3 - y^2$ | (5) |
| 12 | If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$ | (5) |

Module III

Answer any two questions, each carries 5 marks.

- 13 Write the parametric equations of the tangent line to the graph of $\mathbf{r}(t) = \ln t \mathbf{i} + e^{-t} \mathbf{j} + t^4 \mathbf{k}$ at $t = 2$ (5)
- 14 A particle moves along the curve $\mathbf{r} = (t^3 - 4t) \mathbf{i} + (t^2 + 4t) \mathbf{j} + (8t^2 - 3t^3) \mathbf{k}$ where t denotes time. Find (5)
- (i) the scalar tangential and normal components of acceleration at time $t = 2$
- (ii) the vector tangential and normal components of acceleration at time $t = 2$
- 15 Find the equation to the tangent plane and parametric equations of the normal line to the ellipsoid $x^2 + y^2 + 4z^2 = 12$ at the point $(2, 2, 1)$ (5)

Module IV

Answer any two questions, each carries 5 marks.

- 16 Reverse the order of integration and evaluate $\int_0^1 \int_x^1 \frac{x}{x^2 + y^2} dy dx$ (5)
- 17 If R is the region bounded by the parabolas $y = x^2$ and $y^2 = x$ in the first quadrant, evaluate $\iint_R (x + y) dA$ (5)
- 18 Use triple integral to find the volume of the solid bounded by the surface $y = x^2$ and the planes $y + z = 4, z = 0$. (5)

Module V

Answer any three questions, each carries 5 marks.

- 19 If $\mathbf{r} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$ and $r = \|\mathbf{r}\|$, show that $\nabla \log r = \frac{\mathbf{r}}{r^2}$ (5)
- 20 Examine whether $\mathbf{F} = (x^2 - yz) \mathbf{i} + (y^2 - zx) \mathbf{j} + (z^2 - xy) \mathbf{k}$ is a conservative field. If so, find the potential function (5)
- 21 Show that $\nabla^2 f(r) = 2 \frac{f'(r)}{r} + f''(r)$, where $\mathbf{r} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$, $r = \|\mathbf{r}\|$ (5)
- 22 Compute the line integral $\int_C (y^2 dx - x^2 dy)$ along the triangle whose vertices are $(1, 0)$, $(0, 1)$ and $(-1, 0)$ (5)
- 23 Show that the line integral $\int_C (y \sin x dx - \cos x dy)$ is independent of the path and hence evaluate it from $(0, 1)$ and $(\pi, -1)$ (5)

Module VI

Answer any three questions, each carries 5 marks.

- 24 Using Green's theorem, find the work done by the force field $\vec{f}(x, y) = (e^x - y^3) \vec{i} + (\cos y + x^3) \vec{j}$ on a particle that travels once around the unit circle $x^2 + y^2 = 1$ in the counter clockwise direction. (5)
- 25 Using Green's theorem evaluate $\int_C (xy + y^2) dx + x^2 dy$, where c is the boundary of the area common to the curve $y = x^2$ and $y = x$ (5)
- 26 Evaluate the surface integral $\iint_S xz ds$, where S is the part of the plane $x + y + z = 1$ that lies in the first octant (5)
- 27 Using divergence theorem, evaluate $\iint_S \mathbf{F} \cdot \mathbf{n} ds$ where $\mathbf{F} = (x^2 + y) \mathbf{i} + z^2 \mathbf{j} + (e^y - z) \mathbf{k}$ and S is the surface of the rectangular solid bounded by the co ordinate planes and the planes $x = 3, y = 1, z = 3$ (5)
- 28 Apply Stokes's theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = (x^2 - y^2) \mathbf{i} + 2xy \mathbf{j}$ and c is the rectangle in the xy plane bounded by the lines $x = 0, y = 0, x = a$ and $y = b$ (5)
