**A** **B1A005** **Pages: 3**

Reg. No.______________ Name:__________________________

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**

FIRST SEMESTER B.TECH DEGREE EXAMINATION, JUNE/JULY 2017

Course Code**: MA 101**

Course Name**: CALCULUS**

**(For 2015 Admission and 2016 Admission)**

Max. Marks :100 Duration: 3 hours

## PART A

***Answer all questions. Each question carries 5 marks.***

1. (a) Find the interval of convergence and radius of convergence of the infinite series $\sum_{n=0}^{\infty} n!\, x^n$ **(2)**

   (b) Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{5n-1}$ converges or not **(3)**

2. (a) Find the slope of the surface $z = \sqrt{3x + 2y}$ in the $y$-direction at the point $(4,2)$ **(2)**

   (b) Find the derivative of $w = x^2 + y^2$ with respect to $t$ along the path $x = at^2, y = 2at$ **(3)**

3. (a) Find the directional derivative of $f(x,y) = xe^y$ at $(1,1)$ in the direction of the vector $i - j$ **(2)**

   (b) If $\vec{F}(t)$ has a constant direction, then prove that $\vec{F} \times \frac{d\vec{F}}{dt} = 0$ **(3)**

4. (a) Evaluate $\int_0^1 \int_0^1 \frac{1}{\sqrt{(1-x^2)(1-y^2)}}\, dxdy$ **(2)**

   (b) Evaluate $\iint_R \frac{\sin x}{x}\, dxdy$ where $R$ is the triangaular region bounded by the x-axis, $y = x$ and $x = 1$ . **(3)**

5. (a) Show that $\int_A^B (2xy + z^3)\, dx + x^2 dy + 3xz^2 dz$ is independent of the path joining the points A and B. **(2)**

   (b) If $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$, then prove that $\nabla^2 r^n = n(n+1) r^{n-2}$ **(3)**

6. (a) Using line integral evaluate the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ **(2)**

   (b)Evaluate $\int_C (e^x\, dx + 2y\, dy - dz)$ where $C$ is the curve $x^2 + y^2 = 4, z = 2$. **(3)**

## PART B

***Answer any two questions each Module I to IV***

### Module I

7. Determine whether the series converge or diverge $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$ **(5)**

8. Check the absolute convergence or divergence of the series $\sum_{n=1}^{\infty} (-1)^n \frac{(2n-1)!}{3^n}$ **(5)**

9. Find the Taylor series expansion of *log cos x* about the point $\frac{\pi}{3}$ **(5)**

## Module II

10. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, Show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$ **(5)**
11. The length, width and height of a rectangular box are measured with an error of atmost 5%. Use a total differential to estimate the maximum percentage error that results if these quantities are used to calculate the diagonal of the box. **(5)**
12. Locate all relative extrema and saddle points of $f(x,y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$ **(5)**

## Module III

13. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2,-1,2)$ **(5)**
14. Let $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$, then prove that $\nabla f(r) = \frac{f'(r)}{r}\vec{r}$. **(5)**
15. Find an equation of the tangent plane to the ellipsoid $2x^2 + 3y^2 + z^2 = 9$ at the point $(2,1,1)$ and determine the acute angle that this plane makes with the *XY* plane. **(5)**

## Module IV

16. Change the order of integration and hence evaluate $\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$ **(5)**
17. Evaluate $\int_0^2 \int_0^{\sqrt{(4-x^2)}} y(x^2 + y^2)\, dx\, dy$ using polar co-ordinates **(5)**
18. Find the volume of the paraboloid of revolution $x^2 + y^2 = 4z$ cut off by the plane $z = 4$ **(5)**

## Module V

***Answer any 3 questions.***

19. Evaluate the line integral $\int_C (xy + z^3)\, ds$ from $(1,0,0)$ to $(-1,0,\pi)$ along the helix $C$ that is represented by the parametric equations $x = \cos t,\ y = \sin t,\ z = t$ **(5)**
20. Evaluate the line integral $\int_C (y - x)\, dx + x^2 y\, dy$ along the curve C, $y^2 = x^3$ from (1, -1) to (1,1) **(5)**
21. Find the work done by the force field $\vec{F} = (x + y)i + xyj - z^2 k$ along the line segment from $(0,0,0)$ to $(1,3,1)$ and then to $(2,-1,5)$. **(5)**
22. Show that $\vec{F} = (2xy + z^3)i + x^2 j + 3xz^2 k$ is a conservative vector field. Also find its scalar potential. **(5)**
23. Find the values of constants $a,b,c$ so that $\vec{F} = (axy + bz^3)i + (3x^2 - cz)j + (3xz^2 - y)k$ may be irrotational. For these values of $a,b,c$ find the scalar potential of $\vec{F}$ **(5)**

## Module VI

***Answer any 3 questions.***

24. Verify Green's theorem for $\int_C (xy + y^2)dx + x^2\,dy$ where C is bounded by $y = x$ and $y = x^2$ **(5)**

25. Apply Green's theorem to evaluate $\int_C (2x^2 - y^2)dx + (x^2 + y^2)dy$ where C is the boundary of the area enclosed by the x-axis and the upper half of the circle $x^2 + y^2 = a^2$ **(5)**

26. Apply Stokes theorem to evaluate $\int_C (x + y)dx + (2x - y)dy + (y + z)dz$ where C is the boundary of the triangle with vertices $(0,0,0)$, $(2,0,0)$ and $(0,3,0)$ **(5)**

27. Use Divergence theorem to evaluate $\iint_S \vec{F}\cdot\vec{n}\,dS$ where $\vec{F} = xi + zj + yzk$ and S is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = 1$. Also verify this result by computing the surface integral over S **(5)**

28. State Divergence theorem. Also evaluate $\iint_S \vec{F}\cdot\vec{n}\,dS$ where $\vec{F} = axi + byj + czk$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ **(5)**

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