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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FOURTH SEMESTER B.TECH DEGREE EXAMINATION, APRIL 2018

Course Code: MA204
Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS (AE, EC)

Max. Marks: 100 Duration: 3 Hours

(Normal distribution table is allowed in the examination hall)

PART A

Answer any two full questions, each carries 15 marks Marks

1 a) A random variable X has the following probability distribution: (7)

x	-2	-1	0	1	2	3
f(x)	0.1	k	0.2	2k	0.3	3k

Find: i) The value of k ii) Evaluate $P(X < 2)$ and $P(-2 < X < 2)$
iii) Evaluate the mean of X

b) The probability that a component is acceptable is 0.93. Ten components are picked at random. What is the probability that: (8)
i) At least nine are acceptable ii) At most three are acceptable.

2 a) Suppose that the length of a phone call in minutes is an exponential random variable with parameter $\lambda = \frac{1}{10}$. If someone arrives immediately ahead of you at a public telephone booth, find the probability that you will have to wait: (7)
i) More than 10 minutes ii) Between 10 and 20 minutes.

b) For a normally distributed population, 7% of items have their values less than 35 and 89% have their values less than 63. Find the mean and standard deviation of the distribution. (8)

3 a) Fit a binomial distribution to the following data and calculate the theoretical frequencies. (8)

x	0	1	2	3	4	5	6	7	8
f	2	7	13	15	25	16	11	8	3

b) The time between breakdowns of a particular machine follows an exponential distribution, with a mean of 17 days. Calculate the probability that a machine breaks down in a 15 day period. (7)

PART B

Answer any two full questions, each carries 15 marks

4 a) The joint PDF of two continuous random variables X and Y is given by (7)

$$f(x, y) = \begin{cases} kxy & 0 < x < 4,\ 1 < y < 5 \\ 0 & otherwise \end{cases}.$$

Find: i) k ii) The marginal distributions of X and Y
iii) Check whether X and y are independent.

b) A distribution with unknown mean μ has variance equal to 1.5. Use Central Limit Theorem to find how large a sample should be taken from the distribution in order that the probability will be at least 0.95 that the sample mean will be within 0.5 of the population mean. (8)

5 a) The autocorrelation function for a stationary process X(t) is given by $R_{XX}(\tau) = 9 + 2e^{-|\tau|}$. Find the mean value of the random variable $Y = \int_{\tau=0}^{2} X(t)dt$ and the variance of X(t). (7)

b) A random process X(t) is defined by $X(t) = Y(t)\cos(\omega t + \theta)$ Where Y(t) is a WSS process, ω is a constant and $\boldsymbol{\theta}$ is a random variable which is uniformly distributed in [0,2π] and is independent of Y(t). Show that X(t) is WSS. (8)

6 a) Consider the random process $X(t) = A\cos(\omega t + \theta)$ where A and ω are constants and $\boldsymbol{\theta}$ is a uniformly distributed random variable in (0,2π). Check whether or not the process is WSS. (7)

b) The joint PDF of two continuous random variables X and Y is (8)

$$f(x,y) = \begin{cases} 8xy\,, 0 < y < x < 1 \\ 0\,, \quad otherwise \end{cases}$$

i) Check whether X and Y are independent   ii) Find $P(X + Y < 1)$

PART C

Answer any two full questions, each carries 20 marks

7 a) The number of particles emitted by a radioactive source is Poisson distributed. The source emits particles at a rate of 6 per minute. Each emitted particle has a probability of 0.7 of being counted. Find the probability that 11 particles are counted in 4 minutes. (4)

b) Assume that a computer system is in any one of the three states: busy, idle and under repair, respectively, denoted by 0,1,2. Observing its state at 2 P. M. each day, the transition probability matrix is $P = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0 & 0.4 \end{bmatrix}$ (8)

Find out the third step transition probability matrix and determine the limiting probabilities.

c) If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between two consecutive arrivals is: (8)

i) More than 1 minute   ii) Between 1 minute and 2 minutes

iii) Less than or equal to 4minutes.

8 a) Use Trapezoidal rule to evaluate $\int_0^1 x^3\, dx$ considering five subintervals (4)

b) Using Newton's forward interpolation formula, find y at $x = 8$ from the following table: (8)

x :	0	5	10	15	20	25
y :	7	11	14	18	24	32

c) Using Euler's method, solve for y at $x = 0.1$ from $\frac{dy}{dx} = x + y + xy, \quad y(0) = 1$ taking step size $h = 0.025$. (8)

9 a) The transition probability matrix of a Markov chain $\{X_n, n \geq 0\}$ having three states 1, 2 and 3 is $P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.6 & 0.3 \\ 0.4 & 0.3 & 0.3 \end{bmatrix}$ and the initial probability distribution is $p(0) = [\,0.5 \quad 0.3 \quad 0.2\,]$. Find the following: (10)

i) $P\{X_2 = 2\}$ ii) $P\{X_3 = 3, X_2 = 2, X_1 = 1, X_0 = 3\}$.

b) Using Newton-Raphson method, compute the real root of $f(x) = x^3 - 2x - 5$ correct to 5 decimal places. (5)

c) Using Lagrange's interpolation formula, find the values of y when $x = 10$ from the following table : (5)

x :	5	6	9	11
y :	12	13	14	16

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