

Reg No.: \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
THIRD SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2017

**Course Code: CS201**

**Course Name: DISCRETE COMPUTATIONAL STRUCTURES (CS, IT)**

Max. Marks: 100

Duration: 3 Hours

**PART A**

*Answer all questions, each carries 3 marks.*

Marks

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|---|--|-----|
| 1 | Assume $A = \{1, 2, 3\}$ and $\rho(A)$ be its power set. Let $\subseteq$ be the subset relation on the power set. Draw the Hasse diagram of $(\rho(A), \subseteq)$ | (3) |
| 2 | Let $R$ denote a relation on the set of ordered pairs of positive integers such that $(x, y)R(u, v)$ iff $xv = yu$ . Show that $R$ is an equivalence relation      | (3) |
| 3 | Prove that in any group of six people, at least three must be mutual friends or at least three must be mutual strangers.   | (3) |
| 4 | Define GLB and LUB for a partially ordered set. Give an example  | (3) |

**PART B**

*Answer any two full questions, each carries 9 marks.*

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|---|--|-----|
| 5 | a) Suppose $f(x) = x + 2, g(x) = x - 2$ and $h(x) = 3x$ for $x \in \mathbb{R}$ , where $\mathbb{R}$ is the set of real numbers. Find $g \circ f, f \circ g, f \circ f, g \circ g, f \circ h, h \circ g, h \circ h$ and $(f \circ h) \circ g$ | (4) |
|   | b) Prove that every equivalence relation on a set generates a unique partition of the set with the blocks as $R$ -equivalence classes  | (5) |
| 6 | a) Show that the set $\mathbb{N}$ of natural numbers is a semigroup under the operation $x * y = \max(x, y)$ . Is it a monoid?   | (3) |
|   | b) Solve the recurrence relation $a_r + 5a_{r-1} + 6a_{r-2} = 3r^2 - 2r + 1$   | (6) |
| 7 | a) Show that for any commutative monoid $\langle M, * \rangle$ , the set of idempotent elements of $M$ forms a submonoid.  | (5) |
|   | b) Define subsemigroups and submonoids.  | (4) |

**PART C**

*Answer all questions, each carries 3 marks.*

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|----|--|-----|
| 8  | Show that, for an abelian group, $(a * b)^{-1} = a^{-1} * b^{-1}$  | (3) |
| 9  | Show that every chain is a distributive lattice.   | (3) |
| 10 | Simplify the Boolean expression $a'b'c + ab'c + a'b'c'$  | (3) |
| 11 | Let $G = \{1, a, a^2, a^3\}$ ( $a^4 = 1$ ) be a group and $H = \{1, a^2\}$ is a subgroup of $G$ under multiplication. Find all cosets of $H$ . | (3) |

## PART D

*Answer any two full questions, each carries 9 marks.*

- 12 a) Show that the order of a subgroup of a finite group divides the order of the group. (6)  
 b) Define ring homomorphism. (3)
- 13 Show that  $(I, \oplus, \otimes)$  is a commutative ring with identity, where the operations  $\oplus$  and  $\otimes$  are defined, for any  $a, b \in I$ , as  $a \oplus b = a + b - 1$  and  $a \otimes b = a + b - ab$ . (9)
- 14 a) Let  $(L, \leq)$  be a lattice and  $a, b, c, d \in L$ . Prove that if  $a \leq c$  and  $b \leq d$ , then (5)  
 (i)  $a \vee b \leq c \vee d$   
 (ii)  $a \wedge b \leq c \wedge d$   
 b) Show that in a Boolean algebra, for any  $a, b, c$  (4)  

$$(a \wedge b \wedge c) \vee (b \wedge c) = b \wedge c$$

## PART E

*Answer any four full questions, each carries 10 marks.*

- 15 a) a) Construct truth table for  $(\sim p \wedge (\sim q \wedge r)) \vee ((q \wedge r) \vee (p \wedge r))$  (6)  
 b) Explain proof by Contrapositive with example. (4)
- 16 Prove the following implication (10)  
 $(x)(P(x) \vee Q(x)) \implies (x) P(x) \wedge (\exists x) Q(x)$
- 17 a) Represent the following sentences in predicate logic using quantifiers (6)  
 (i) "x is the father of the mother of y"  
 (ii) "Everybody loves a lover"  
 b) Determine whether the conclusion C follows logically from the premises (4)  
 $H_1: \sim p \vee q, H_2: \sim(q \wedge \sim r), H_3: \sim r$  C:  $\sim p$
- 18 a) Without using truth table prove  $p \rightarrow (q \rightarrow p) \iff \sim p \rightarrow (p \rightarrow q)$  (4)  
 b) Determine the validity of the following statements using rule CP. (6)  
 "my father praises me only if I can be proud of myself. Either I do well in sports or I can't be proud of myself. If I study hard, then I can't do well in sports. Therefore if my father praises me then I do not study well"
- 19 a) Show that  $r \rightarrow s$  can be derived from the premises  $p \rightarrow (q \rightarrow s), \sim r \vee p, q$  (4)  
 b) Prove, by Mathematical Induction, that  $n(n+1)(n+2)(n+3)$  is divisible by 24, for all natural numbers n (6)
- 20 a) "If there are meeting, then travelling was difficult. If they arrived on time, then travelling was not difficult. They arrived on time. There was no meeting". Show that these statements constitute a valid argument. (6)  
 b) Show that  $2^n < n!$  For  $n \geq 4$  (4)

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