

Reg. No. _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
THIRD SEMESTER B.TECH DEGREE EXAMINATION, JULY 2017

Course Code: **MA 201**Course Name: **LINEAR ALGEBRA AND COMPLEX ANALYSIS.**

Max. Marks :100

Duration: 3 hours

PART A*Answer any two questions.*

1. (a) Does the limit $\lim_{z \rightarrow 0} \frac{z}{z}$ exist? If yes find the value. If no, explain why? (8)
(b) If $f(z) = u + iv$ is analytic, prove that $u = \text{constant}$ and $v = \text{constant}$ are families of curves cutting orthogonally (7)
2. (a) Find the image of the semi-circle $y = +\sqrt{4 - x^2}$ under the transformation $w = z^2$ (7)
(b) Find the image of the half-plane $\text{Re}(z) \geq 2$ under the map $w = iz$ (8)
3. (a) Find the points, if any, in complex plane where the function $f(z) = 2x^2 + y + i(y^2 - x)$ is
(i) differentiable (ii) analytic. (8)
(b) Prove that the function $u(x, y) = x^3 - 3xy^2 - 5y$ is harmonic everywhere. Also find the harmonic conjugate of u . (7)

PART B*Answer any two questions.*

4. (a) Evaluate $\int_C \bar{z} dz$ where C is given by $x = 3t, y = t^2, -1 \leq t \leq 4$. (8)
(b) Show that $\int_C (2 + z)^2 dz = -\frac{i}{3}$ where C is any path connecting the points -2 and $-2 + i$ (7)
5. (a) Evaluate $\int_C \frac{5z+7}{z^2+2z-3} dz$ where C is the circle $|z - 2| = 2$. (8)
(b) Find the Laurent's series expansion of $\frac{1}{z-z^3}$ in $1 < |z + 1| < 2$. (7)
6. (a) Use Cauchy's integral formula to evaluate $\int_C \frac{z+1}{z^4+2iz^3} dz$ where C is $|z| = 1$. (8)
(b) Using Contour integration, evaluate $\int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10x^2+9} dx$ (7)

PART C

Answer any two questions.

7. (a) Using Gauss elimination method, find the solution of the system of equations
 $x + 2y - z = 3$, $3x - y + 2z = 1$, $2x - 2y + 3z = 2$ and $x - y + z = -1$ (7)
- (b) Find the values of μ for which the system of equations $x + y + z = 1$, $x + 2y + 3z = \mu$ and $x + 5y + 9z = \mu^2$ will be consistent. For each value of μ obtained, find the solution of the system. (7)
- (c) Prove that the vectors $(2,3,0)$, $(1,2,0)$ and $(8,13,0)$ are linearly dependent in R^3 . (6)
8. (a) Find the rank of the matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ (7)
- (b) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ (7)
- (c) Write the canonical form of the quadratic form $Q(x, y, z) = 3x^2 + 5y^2 + 3z^2 - 2xy + 2xz - 2yz$ and hence show that $Q(x, y, z) > 0$ for all non-zero values of x, y, z . (6)
9. (a) Diagonalize the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ and hence find A^4 . (7)
- (b) If 2 is an eigen value of $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$, without using its characteristic equation, find the other eigen values. Also find the eigen values of $A^3, A^T, A^{-1}, 5A, A - 3I$ and $\text{adj } A$. (7)
- (c) Show that $17x^2 - 30xy + 17y^2 = 128$ represents an ellipse. Also find the equations of the major and minor axes of the ellipse in terms of x and y . (6)
