Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

THIRD SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2018

Course Code: MA201

Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer any two full questions, each carries 15 marks Marks

1	a)	Prove that $f(z) = e^{x+iy}$ is analytic. Find $f'(z)$.	(7)
	b)	Show that $v = 3x^2y - y^3$ is harmonic. Also find the harmonic conjugate of v.	(8)
2	a)	Find the linear fractional transformation that maps $z_1 = 0, z_2 = 1, z_3 = \infty$ onto $w_1 = -1, w_2 = -i, w_3 = 1$ respectively.	(8)
	b)	Find the image of the lines $x = a$ and $y = b$ where a and b are constants, under the transformation $w = z^2$	(7)
3	a)	If $f(z) = u + iv$ is analytic, prove that $u = c_1$ and $v = c_2$ are families of curves cutting orthogonally.	(7)
	b)	Prove that $w = \frac{z-i}{1-iz}$ maps the upper half plane (y>0) into the interior of $\|w\| = 1$	(8)

PART B

Answer any two full questions, each carries 15 marks

4	a)	Expand $f(z) = \frac{1}{z^2}$ as Taylor's series about $z = 2$	(7)
	b)	Evaluate $\oint_C \frac{\cos \pi z^2 + \sin \pi z^2}{(z-1)(z-2)} dz$ where C is $\|z\| = 3$, using Cauchy's integral formula.	(8)
5	a)	Evaluate $\oint_C \frac{z-23}{z^2-4z-5} dz$ where $C: \|z-2-i\| = 3.2$, using Cauchy's residue theorem.	(7)
	b)	Show that $\int_{-\infty}^{\infty} \frac{1}{(1+x^2)^3} dx = \frac{3\pi}{8}$	(8)
6	a)	Find the Laurent's series expansion of $f(z) = \frac{e^{2z}}{(z+1)^3}$ about $z = -1$	(7)
	b)	Find the poles and residues of the function $f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}$	(8)

PART C

Answer any two full questions, each carries 20 marks

7 a) Find the Eigen value and Eigen vector of the matrix A=$\begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$ (8)

b) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \\ 8 & 1 & 14 & 17 \end{bmatrix}$ (6)

c) Solve the system of equations $x - y + z = 0, -x + y - z = 0,$ (6)

$10y + 25z = 90\ , 20x + 10y = 80$

8 a) Find out what type of conic section the quadratic form (8)

$q = 3x_1^2 + 21x_1x_2 + 3x_2^2 = 0$ represents.

b) Show that the matrix A = $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & \sqrt{3} \\ 0 & \sqrt{3} & 1 \end{bmatrix}$ is orthogonal. (6)

c) Show that the system of equations are inconsistent. (6)

$2x + 6y = -11, \quad 6x + 20y - 6z = -3, \quad 6y - 18z = -1$

9 a) (i) Show that the vectors $(1,-1,0), (1,3,-1)$ and $(5,3,-2)$ are linearly dependent. (8)

(ii) Show that the quadratic form $4x_1^2 + 12x_1x_2 + 13x_2^2 = 16$ is positive definite.

b) Diagonalize the matrix A=$\begin{bmatrix} 6 & 0 & 0 \\ 12 & 2 & 0 \\ 21 & -6 & 9 \end{bmatrix}$ (12)

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